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# **Zumo Segway Documentation**

*Release 0.0.1*

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# CHAPTER 1

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## Introduction

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This documentation features the implementation of the Segway behavior in a Zumo32u4 board. The process started with the modeling of the system according to the capabilities of the Zumo32u4. Therefore, a *Zumo32u4* chapter was written with the relevant about the Zumo's IMU, Zumo's encoders and Zumo's motos.

Secondly a model from the literature was selected. The model in [3] was firstly analyzed. The developed model in [3] was develop upon simplifying and considering it as a cart with an inverted pendulum installed on top. It defined the input as the force applied to the cart. To use this model the force had to be written in terms of the torque applied to the pendulum.

The model in [15] proved to be better applicable for implementation on a Zumo32u4 since it was in already in terms of torque applied to the pendulum system and the physical model upon the state variable model was based could be easily mapped to the Zumo by simplifying it.

None of the model in [3] and [15] considered a caterpillar driven system. Nevertheless, [15] considers explicitly the inertia moment of the wheels which can be replaced by the inertia moment of the caterpillar system.

*Segway Model* chapter presents how the model of [15] was adapted to the Zumo32u4 considering also actuator's scaling factors.

Later, in *LQR Controller Design* chapter the design of the control law using LQR method is presented. The design was done based on the design done in [3] and [15].

After the designing the control law the *Controller Implementation* chapter explains how the controller was implemented.

As expected [3] nor [15] present a method to obtain accurate friction coefficients, what makes the model also inaccurate. But the inaccuracy of the model lead to unstable controlled system. Which showed the need of add a scale factor in the controlling law.

The online version of this documentation can be seen at <http://zumosegway.readthedocs.io/en/latest/index.html> and the source code documented here is hosted as an opensource project on github at <https://github.com/pjcuadra/zumosegway>.



The Zumo32U4 is a hardware development platform includes a built-in Arduino-compatible ATmega32U4 microcontroller, an LCD, encoders for closed-loop motor control, and proximity sensors for obstacle detection. It's high-performance motors and integrated sensors make it versatile enough to serve as a general-purpose small robot. [1]. Fig. 2.1 shows the Zumo32u4 Robot.

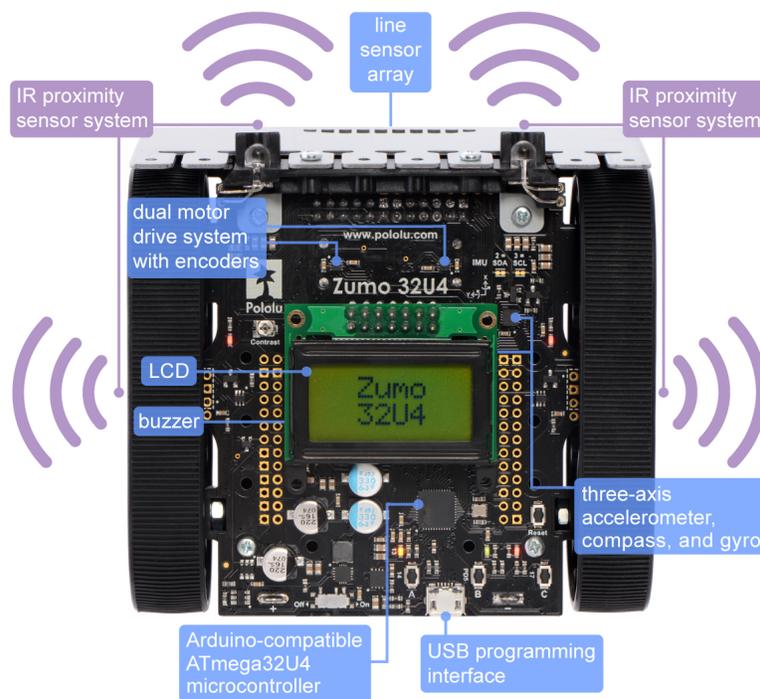


Fig. 2.1: Zumo32u4 Robot [1]

For the Segway implementation the Zumo32u4's blades, line sensor array and top most IR Proximity sensors were

removed because the Zumo32u4 balances on the edge where the blade is (top edge in Fig. 2.1). The opposite edge can't be used because the battery (not visible in the Fig. 2.1) holder completely sits on the floor.

At the time of writing this documentation there are three different Zumo32u4 robots;

- Zumo32u4 robot with 50:1 HP motors
- Zumo32u4 robot with 75:1 HP motors
- Zumo32u4 robot with 100:1 HP motors

The main difference between the available configurations is the gear ration of the motors. For this project the Zumo32u4 robot with 100:1 HP motors was used.

In the following subsections the relevant Zumo32u4 components will be described in more detail.

## 2.1 Inertial Management Unit

The Zumo32U4 includes on-board inertial sensors that can be used in advanced applications, such as helping our Zumo detect collisions and determine its own orientation by implementing an inertial measurement unit (IMU).

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**Note:** We used *IMU* as the main sensory-model for *Segway*.

---

We took the aid of following sensor components of *IMU*;

- Gyroscope : *ST L3GD20H* 3-axis gyroscope.
- Accelerometer : *ST LSM303D* compass module, which combines a 3-axis accelerometer and 3-axis magnetometer.

---

**Note:**

- Both sensor chips share an  $I^2C$  bus connected to the ATmega32U4's  $I^2C$  interface.
  - Level shifters built into the main board allow the inertial sensors, which operate at 3.3 V, to be connected to the ATmega32U4 (operating at 5 V).
- 

### 2.1.1 Gyroscope

We consider the following aspects of Gyroscope for *IMU* sensory-model;

- Gyroscope provides the change in orientation of the Zumo (Roll, Yaw, Pitch). Integration of result provides the position details.
- *ST L3GD20H* Gyroscope operation is based on angular momentum.
- *ST L3GD20H* provides; \* Selectable full-scale range of  $\pm 245dps/\pm 500dps/\pm 2000dps$ , with the  $8.75 \frac{mdps}{digit}$  /  $17.5 \frac{mdps}{digit}$  /  $70 \frac{mdps}{digit}$  sensitivity, respectively.
  - Selectable data sampling rate.
  - Low-Pass filter to reduce noise with selectable cut-off frequencies.

Listing 2.1: Gyroscope configuration [9]

```
// Set up the L3GD20H gyro.
gyro.init();

// 800 Hz output data rate,
// low-pass filter cutoff 100 Hz.
gyro.writeReg(L3G::CTRL1, 0b11111010);

// 2000 dps full scale.
gyro.writeReg(L3G::CTRL4, 0b00100000);

// High-pass filter disabled.
gyro.writeReg(L3G::CTRL5, 0b00000000);
```

**Note:** All other register were left with their default value. Review [13] for more information regarding default values.

A more detailed descriptions of the configuration used is shown below;

- `L3G::CTRL1.DR[1:0] = 0x3` selects the  $800Hz$  data sampling rate [13].
- `L3G::CTRL1.BW[1:0] = 0x3` selects  $100Hz$  gyroscope data cut-off frequency [13].
- `L3G::CTRL1.PD = 0x1` selects the normal mode disabling power mode, so the signal will be always be sampled [13].
- `L3G::CTRL1.XEN = 0x0`, `L3G::CTRL1.YEN = 0x1` and `L3G::CTRL1.ZEN = 0x0` enables only the needed gyroscope channel.
- `L3G::CTRL4.FS[1:0] = 0x2` selects the full-scale of  $\pm 2000dps$  with a sensitivity of  $70mdps/digit$  [13].
- `L3G::CTRL5.HPen = 0x0` disable the High-Pass filter [13].

## 2.1.2 Accelerometer

We consider the following aspects of Accelerometer for *IMU* sensory-model;

- *ST LSM303D* Accelerometer provides the linear acceleration based on vibration.
- By virtue of linear acceleration, Accelerometer provides 3-dimensional position (X-,Y-,Z- axis). [14]
- *ST LSM303D* provides  $\pm 2/\pm 4/\pm 6/\pm 8/\pm 16$  selectable linear acceleration full-scale. [14]
- *ST LSM303D* provides  $3.125Hz/6.25Hz/12.5Hz/25Hz/50Hz/100Hz/200Hz/400Hz/800Hz/1600Hz$  selectable sampling rate. [14]

For the implementation of the Segway the sampling frequency,  $f_s = 50Hz$ , and full-scale range,  $acc_{range} = \pm 8g$ , were selected. Therefore, the *ST LSM303D* configuration code is shown in Listing 2.2

Listing 2.2: Compass configuration [9]

```
// Set up the LSM303D accelerometer.
compass.init();

// 50 Hz output data rate
compass.writeReg(LSM303::CTRL1, 0x57);
```

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(continued from previous page)

```
// 8 g full-scale
compass.writeReg(LSM303::CTRL2, 0x18);
```

**Note:** All other register were left with their default value. Review [14] for more information regarding default values.

A more detailed descriptions of the configuration used is shown below;

- `LSM303::CTRL1.AODR[3:0] = 0x5` sets the  $f_s = 50Hz$ . [14]
- `LSM303::CTRL1.BDU = 0x1` enables atomic update for the acceleration read register. Meaning that the entire register will be written at once [14].
- `LSM303::CTRL1.AXEN = 0x1`, `LSM303::CTRL1.AYEN = 0x1` and `LSM303::CTRL1.AZEN = 0x1` enables all three acceleration channels. [14]. All three are needed because the magnitude of the acceleration vector is calculated to filter some measurement noise. Listing 2.4 shows how the magnitude is used to filter the noise.
- `LSM303::CTRL2.AFS[2:0] = 0x3` sets  $acc_{range} = \pm 8g$ .

### 2.1.3 Combine Gyroscope and Accelerometer

Gyroscope gives angular position but has tendency to drift over the period of time. Accelerometer gives Inertia, and ultimately position but it is slow. Hence, Accelerometer output is used to correct position obtained from Gyroscope on periodic interval of time.

First the Gyroscope is being sampled as frequently as possible. Then the data of the Gyroscope is integrated and to give the current Zumo32u4's angle as fast as possible. Listing 2.3 shows how the sampling and integration was performed;

Listing 2.3: Gyroscope angle sampling and integration [9]

```
/** Zumos Gyro */
L3G gyro;

/**
 * Reads the Gyro changing rate and integrate it adding it to the angle
 */
void sampleGyro() {
    // Figure out how much time has passed since the last update.
    static uint16_t lastUpdate = 0;
    uint16_t m = micros();
    uint16_t dt = m - lastUpdate;
    float gyroAngularSpeed = 0;
    lastUpdate = m;

    gyro.read();
    // Obtain the angular speed out of the gyro. The gyro's
    // sensitivity is 0.07 dps per digit.
    gyroAngularSpeed = ((float)gyroOffsetY - (float)gyro.g.y) * 70 / 1000.0;

    // Calculate how much the angle has changed, in degrees, and
    // add it to our estimation of the current angle.
    angularPosition += gyroAngularSpeed * dt / 1000000.0;
}
```

The selected sampling frequency for all sensors was  $f_s = 50\text{Hz}$  meaning that every  $20\text{ms}$  the integrated angle from the gyroscope is corrected with the angle given by the Accelerometer. Listing 2.4 shows how the correction is performed.

Listing 2.4: Integrated gyroscope angle correction with accelerometer's angle [9]

```

/** Zumos Accelerometer */
LSM303 compass;

/**
 * Read the acceleormeter and adjust the angle
 */
void sampleAccelerometer() {
    static uint16_t lastUpdate = 0;
    uint16_t m = micros();
    uint16_t dt = m - lastUpdate;
    float gyroAngularSpeed = 0;

    lastUpdate = m;

    compass.read();
    accelerometerAngle = atan2(compass.a.z, -compass.a.x) * 180 / M_PI;

    // Calculate the magnitude of the measured acceleration vector,
    // in units of g.
    LSM303::vector<float> const aInG = {
        (float) compass.a.x / 4096,
        (float) compass.a.y / 4096,
        (float) compass.a.z / 4096
    };
    float mag = sqrt(LSM303::vector_dot(&aInG, &aInG));

    // Calculate how much weight we should give to the
    // accelerometer reading. When the magnitude is not close to
    // 1 g, we trust it less because it is being influenced by
    // non-gravity accelerations, so we give it a lower weight.
    float weight = 1 - 5 * abs(1 - mag);
    weight = constrain(weight, 0, 1);
    weight /= 10;

    // Adjust the angle estimation. The higher the weight, the
    // more the angle gets adjusted.
    angularPosition = weight * accelerometerAngle + (1 - weight) * angularPosition;
    angularSpeed = (angularPosition - prevAngularPosition) * 1000000.0 / dt;
    prevAngularPosition = angularPosition;
}

```

#### Note:

- Note that `angularPosition` is derivated to get `angularSpeed`, because both quantities are needed by the state variable model used. For more information review the [State Variable Model](#).
- The sign of the angle has been changed from the one in the original balancing example [9] to match our reference framework.

- `src/SegwayLQR/ZumoIMU.ino` holds the source code that handles the IMU.
- 

**Warning:** All angles are given in degrees because during implementation it was proved that it was easier to catch bugs if the angle was in degrees. One reason for this was that degrees are scaled up with respect with radians it was easier to catch integer divisions causing the angle to be zero. Furthermore the use of degrees is a little more intuitive than radians.

### 2.1.4 ZumoIMU API

**class ZumoIMU**

float **accelerometerAngle** = 0  
Accelerometer angle

L3G **gyro**  
Zumo's Gyro

LSM303 **compass**  
Zumo'ss Accelerometer

float **gyroOffsetY**  
Gyro's bias

float **prevAngularPosition** = 0  
Previous Angular position

void **setupIMU** ()  
Setup the Gyro and Accelerometer

void **sampleGyro** ()  
Reads the Gyro changing rate and integrate it adding it to the angle

void **sampleAccelerometer** ()  
Read the accelerometer and adjust the angle

void **calibrateGyro** ()  
Calibrate the Gyroscope. Get the bias.

## 2.2 Motors

The Zumo32U4 includes a *DRV8837* [2] as a dual motor driver. The *DRV8837* handles the current requirements for motors. It also provides rotational direction control of the motors.

The Zumo Library [8] provides the following API to control the motors' speed.

**class Zumo32U4Motors**

Controls motor speed and direction on the Zumo 32U4.

void **setSpeeds** (int16\_t *leftSpeed*, int16\_t *rightSpeed*)  
Sets the speeds for both motors.

int16\_t **leftSpeed**  
A number from -400 to 400 representing the speed and direction of the left motor. Values of -400 or less result in full speed reverse, and values of 400 or more result in full speed forward.

**int16\_t rightSpeed**

A number from -400 to 400 representing the speed and direction of the right motor. Values of -400 or less result in full speed reverse, and values of 400 or more result in full speed forward.

`Zumo32U4Motors::setSpeeds()` enables us to control both motors. For instance a 400 value in `leftSpeed` will set the a 100% duty cycle with level 6V PWM between the left motor's *DRV8837*'s outputs (OUT1 and OUT2). Which results in applying full forward speed to the left motor. A value of -200 will set a 50% duty cycle with level of -6V PWM between the left motor's *DRV8837*'s outputs (OUT1 and OUT2).

The PWM duty cycle can be translated into a percentage of maximum current applied to the motor  $I_{max}$ . Table 2.1's shows the value of  $I_{max}$  which corresponds to the Stall Current at 6V. Additionally, the torque of the motor is linearly related to the current is applied to it. Therefore, the torque of the motors can be calculated by;

$$\tau_0 = \frac{\tau_s}{400} \times speed_{PWM} \quad (2.1)$$

Where  $\tau_s$  is the stall torque of the motor at 6V. In our case we used  $\tau_t = 0.211846Nm$ .

Table 2.1: Motors' Torque related information per Zumo32u4 model [11]

| Model | Stall Torque @ 6V (Nm) | Free-run Speed @ 6V (RPM) | Stall Current @ 6V (A) |
|-------|------------------------|---------------------------|------------------------|
| 75:1  | 0.105923               | 625                       | 1.6                    |
| 50:1  | 0.155354               | 400                       | 1.6                    |
| 100:1 | 0.211846               | 320                       | 1.6                    |

## 2.3 Encoders

The Zumo32U4 includes on-board encoders for closed-loop motor control. In our application we need to read the angular position and angular speed of the motors because to the Segway model we used defines these quantities as state variables. For more information review the [State Variable Model](#).

The optical encoder [7] available in the Zumo32u4 uses the *Sharp GP2S60* [12]. In the case of the encoders the Zumo Library [8] abstracts all the needed configuration and read/write operations. The only thing needed for implementation is the interpretation of the count provided by the encounters.

According to [7] the optical encoder provides 12 CPR (counts per revolution). Therefore its count can be interpreted as shown in (2.2).

$$cycles = \frac{encodersCount}{12 \times gearRatio} \quad (2.2)$$

According to [5], [6] and [4] the gear ratio for the different Zumo modules are shown in Table 2.2.

Table 2.2: Gear Ratio and Count to Degree per Zumo32u4 model

| Model | Gear Ratio | Count to Degree (°) |
|-------|------------|---------------------|
| 50:1  | 51.45      | 0.58309°            |
| 75:1  | 75.81      | 0.39573°            |
| 100:1 | 100.37     | 0.29889°            |

Furthermore we can convert number of cycles to degree with the conversion ratio  $\frac{360^\circ}{1 \times cycle}$ . Multiplying (2.2) by this ratio;

$$motorAngularPosition = encodersCount \times countToDegrees$$

With;

$$countToDegrees = \frac{360}{12 \times gearRatio}$$

**Note:** Since the Zumo32u4 has one encoder per motor we decided to estimate the actual motor: *motorAngularPosition* as the average of both motors' angular position.

---

Listing 2.5 shows the implementation of the explained above. Note that the *motorAngularSpeed* is obtained by derivating *motorAngularPosition* both are needed by our state variable model. For more information review the *State Variable Model*.

Listing 2.5: Encoders Code

```
/** Zumo 100:1 motor gear ratio */
const float gearRatio = 100.37;
/** Encoder count to cycle conversion constant */
const float countToDegrees = 360 / (float)(12.0 * gearRatio);

/** Zumo encoders */
Zumo32U4Encoders encoders;

/**
 * Clear the counters of the encoder
 */
void clearEncoders() {
    encoders.getCountsAndResetLeft();
    encoders.getCountsAndResetRight();
}

/**
 * Sample the encoders
 */
void sampleEncoders() {
    static float prevPosition = 0;
    static uint16_t lastUpdate = 0;
    static float leftPosition = 0;
    static float rightPosition = 0;
    uint16_t m = micros();
    uint16_t dt = m - lastUpdate;
    lastUpdate = m;

    leftPosition += (float)encoders.getCountsAndResetLeft() * countToDegrees;
    rightPosition += (float)encoders.getCountsAndResetRight() * countToDegrees;
    float motorAngularPosition = -(leftPosition + rightPosition) / 2.0;

    motorAngularSpeed = (motorAngularPosition - prevPosition) * 1000000.0 / dt;
    prevPosition = motorAngularPosition;
}
```

---

**Note:**

- `encoders.getCountsAndResetLeft()` and `encoders.getCountsAndResetRight()` get the actual count of the respective motor and clear its counter.
  - *motorAngularPosition* is the average of both speeds multiplied by  $-1$  to match our reference frame.
  - The source code of the Encoders can be reviewed at <src/SegwayLQR/ZumoEncoders.ino>
-

### 2.3.1 ZumoEncoders API

**class ZumoEncoders**

**const float gearRatio** = 100.37

Zumo 100:1 motor gear ratio

**const float countToDegrees** = 360 / (float)(12.0 \* gearRatio);

Encoder count to cycle conversion constant

Zumo32U4Encoders **encoders**

Zumo encoders

void **clearEncoders** ()

Clear the counters of the encoder

void **sampleEncoders** ()

Sample the encoders



## Segway Model

The model of the balancing robot proposed in [15] is derived from the physical description of Fig. 3.1.

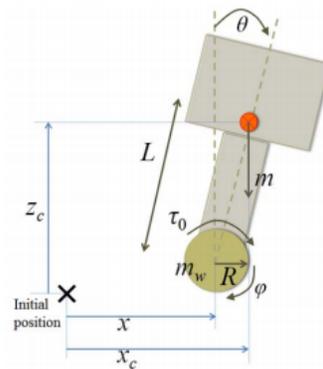


Fig. 3.1: Segway's relevant parameters [15]

Where;

- $x$  is the horizontal position of the center of the wheel.
- $\varphi$  is the clockwise rotation angle of the wheel from the horizontal axis
- $\theta$  is the clockwise rotation angle of the Zumo32u4 from the horizontal axis
- $m$  is the mass of the entire robot
- $m_w$  is the mass of the wheel
- $R$  radius of the wheel
- $L$  length between the center of the wheel and the COM (center of mass)
- $\tau_0$  is the applied torque
- $I$  inertia of the body part

- $I_w$  inertia of the wheel

### 3.1 System's dynamics equations

The obtained differential equations of the system in [15] are;

$$E \begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \end{bmatrix} + F \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \end{bmatrix} + G\theta = H\tau_0 \quad (3.1)$$

With,

$$E = \begin{bmatrix} I_w + (m_w + m)R^2 & mRL \\ mRL & I + mL^2 \end{bmatrix}$$

$$F = \begin{bmatrix} \beta_\gamma + \beta_m & -\beta_m \\ -\beta_m & \beta_m \end{bmatrix}$$

$$G = \begin{bmatrix} 0 \\ -mgL \end{bmatrix}$$

$$H = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

### 3.2 Model Adaptation

In [15] they define  $L$  as in (3.2) with the variables define as in Fig. 3.2.

$$L = \frac{L_2}{2} + \frac{L_1 + L_2}{2} \frac{m_1}{m} \quad (3.2)$$

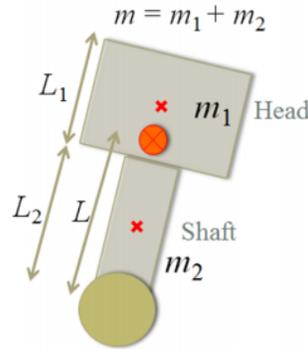


Fig. 3.2: Center of Mass calculation [15]

Similarly [15] defines the inertia momentum of the robot as in (3.3).

$$I = m_1 \left( \frac{L_1}{2} + L_2 \right)^2 + \frac{1}{12} m_2 L_2^2 \quad (3.3)$$

Given the geometry of the Zumo32u4 we consider that  $m_1 = 0$  and  $L_1 = 0$ . Therefore the distance to the COM and the inertia momentum can be calculated  $L = \frac{L_2}{2}$  and  $I = \frac{1}{12} m_2 L_2^2$ , respectively.

Furthermore, the model in [15] consider a normal wheel. In our Zumo32u4 we have a caterpillar system. (3.4) shows how the inertia moment of the caterpillar system was calculated.

$$I_w = I_{w_1} + I_c + I_{w_2} \quad (3.4)$$

Where the  $I_{w_1}$  and  $I_{w_2}$  are the inertia moment of the both wheels and  $I_c$  is the inertia moment of the caterpillar band. In our case both wheels are equal and can be calculated as in (3.5)

$$I_{w_i} = m_w R^2 \quad (3.5)$$

Additionally the inertia of the caterpillar band can be calculated as shown in (3.6). Where  $m_c$  is the mass of the caterpillar band.

$$I_c = m_c R^2 \quad (3.6)$$

Finally the inertia moment of the entire caterpillar system can be calculated as in (3.7).

$$I_w = (2 \cdot m_w + m_c) R^2 \quad (3.7)$$

### 3.3 Input Adaptation

The model in [15] defines the input to be the torque  $\tau_0$ . Since the actual input to our system is the PWM applied to the motors we can use the equation defined in the subsection Motors of the chapter Zumo32u4, shown in (3.8).

$$\tau_0 = \frac{\tau_s}{400} \times speed_{PWM} \quad (3.8)$$

Merging (3.8) and (3.1) we obtain;

$$E \begin{bmatrix} \ddot{\varphi} \\ \ddot{\theta} \end{bmatrix} + F \begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \end{bmatrix} + G\theta = H_1 speed_{PWM}$$

With;

$$H_1 = H * \frac{\tau_s}{400}$$

### 3.4 State Variable Model

Finally the state variable model of the system can be calculated as shown in (3.9).

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + D \end{aligned} \quad (3.9)$$

With the state variable vector;

$$x = \begin{bmatrix} \varphi \\ \theta \\ \dot{\varphi} \\ \dot{\theta} \end{bmatrix}$$

And the constant matrices;

$$A = \left[ \begin{array}{cc|cc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline 0 & -E^{-1}G & -E^{-1}F & \end{array} \right]$$

$$B = \begin{bmatrix} 0 \\ 0 \\ -E^{-1}H \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

---

## LQR Controller Design

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The controller to be implemented is a full-state feedback controller. The LQR controller was selected. Fig. 4.1 shows the block diagram of the entire system to be implemented.

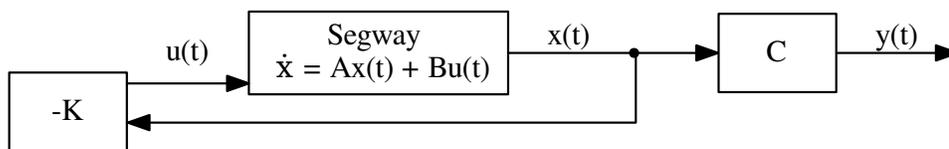


Fig. 4.1: Full-state feedback block diagram

### 4.1 Physical parameters

All the parameters needed for the model can be seen in [Listing 4.1](#)

Listing 4.1: Physical constant script *scripts/load\_physical\_constants.m*  
[1]

```
% Sampling constants
T_s = 20e-3;
f_s = 1/T_s;

% Constants (context)
```

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```

m = 0.24200; % Mass of the zumo
m_1 = 0;
m_2 = m;
L_1 = 0;
L_2 = 0.062;
L = L_2/2 + (L_1 + L_2) * m_1/(2*m); % Height of the zumo
beta_m = 0.01;
beta_gamma = 0.01;

g = 9.8100; % Gravitational constant
R = 0.019; % Wheel radius

I = m_1*(L_1/2 + L_2)^2 + (m_2*L_2^2)/12; % Inertial momentum

m_w = 0.004; % Mass of the wheel
m_c = 0.009; % Mass of the caterpillar band

I_w_i = m_w*R^2; % Inertia momentum of wheels
I_c = m_c*R^2; % Inertia momentum of Caterpillar band
I_w = 2*I_w_i + I_c; % Inertia momentum of Caterpillar system

% Motor's constants
motor_stall_torque = 0.211846554999999; % According to specs 30 oz-in
pulse2torque = motor_stall_torque/400;

```

**Note:** For  $\beta_m$  and  $\beta_\gamma$  are set to a dummy value as in [15].

## 4.2 Model

To setup the model the script in Listing 4.2 was used.

Listing 4.2: Get State Variable Model script `scripts/get_ssmodel.m` [1]

```

function model = get_ssmodel()

% Constants (context)
load_physical_constants

E = [(I_w + (m_w + m)*R^2) m*R*L;
     m*R*L (I + m*L^2)];

F = [(beta_gamma + beta_m) -beta_m;
     -beta_m beta_m];

G = [0; -m*g*L];

H_1 = [1; -1] * pulse2torque;

states = size(E, 1);

A = [zeros(states) eye(states);
     zeros(states, 1) -inv(E)*G -inv(E)*F];

```

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```

B = [zeros(states, 1);
      -inv(E)*H_1];
C = [1 0 0 0;
      0 1 0 0;
      0 0 1 0;
      0 0 0 1];
D = [0; 0; 0; 0];

model = ss(A, B, C, D);

end

```

A second script, shown in Listing 4.3 was also added to get the model that also obtains the transfer function  $H(s) = \frac{\Theta(s)}{S(s)}$ . Where,  $S(s)$  is the Laplace transform of the  $speed_{PWM}$  function. This transfer function was used to further analysis not presented in this documentation.

Listing 4.3: Get Model script `scripts/get_model.m` [1]

```

function [plant, model] = get_model()

% Get the state variable model
model = get_ssmode();

% Get the transfer function
plant = tf(model);
plant = plant(2);

end

```

## 4.3 Controllability

Before the actually designing the controller we need to check it's controllability. The controllability check done can be seen in Listing 4.4.

Listing 4.4: Check controllability `scripts/lqr_design.m` [1]

```

% Check controllability
co = ctrb(model);
if (rank(co) > n_states)
    disp(" -> Error! System isn't controllable");
    return;
else
    disp(" -> Great! System is controllable");
end

```

## 4.4 Observability

Similarly, the system's observability has to be also verified. This verification is shown in Listing 4.5.

Listing 4.5: Check Observability *scripts/lqr\_design.m* [1]

```
ob = obsv(model);  
if (rank(ob) > n_states)  
    disp(" -> Error! System isn't observable");  
    return;  
else  
    disp(" -> Great! System is observable");  
end
```

## 4.5 Control Law

To obtain the control law  $K$  the script in Listing 4.6.

Listing 4.6: Control Law Calculation *scripts/lqr\_design.m* [1]

```
Q = eye(size(model.a, 1));  
  
R = 1  
  
[K, X, P] = lqr(model, Q, R);  
  
K_s = K*pi/180;
```

---

**Note:**

- As in [15] equally weighted states and outputs were used. Therefore,  $Q = I$  and  $R = 1$ . Where,  $I$  is an identity matrix with the size of  $A$ .
  - A scaled control law  $K_s$  is also calculated. The scale factor is needed because the angles measured by accelerometer/gyro and encoders was done in degrees.
- 

The obtained control law is shown in (4.1). And the scaled version in (4.2)

$$K = [1 \quad 483.6133 \quad 10.0038 \quad 20.3053] \quad (4.1)$$

$$K_s = [0.0175 \quad 8.4406 \quad 0.1746 \quad 0.3544] \quad (4.2)$$

## 4.6 Controller Simulation

After designing the control law the controller is simulated as shown in Listing 4.7.

Listing 4.7: Controller simulation excerpt *scripts/lqr\_design.m* [1]

```
Ac = model.a - model.b*K;  
sys_cl = ss(Ac, model.b, model.c, model.d);  
figure(1);  
clf(1)  
impulse(sys_cl);
```

The simulation results can be seen in Fig. 4.2. Since the  $\frac{d\varphi}{dt}$  and  $\frac{d\theta}{dt}$  are faster variables a zoomed result simulation result can also be seen in Fig. 4.3.

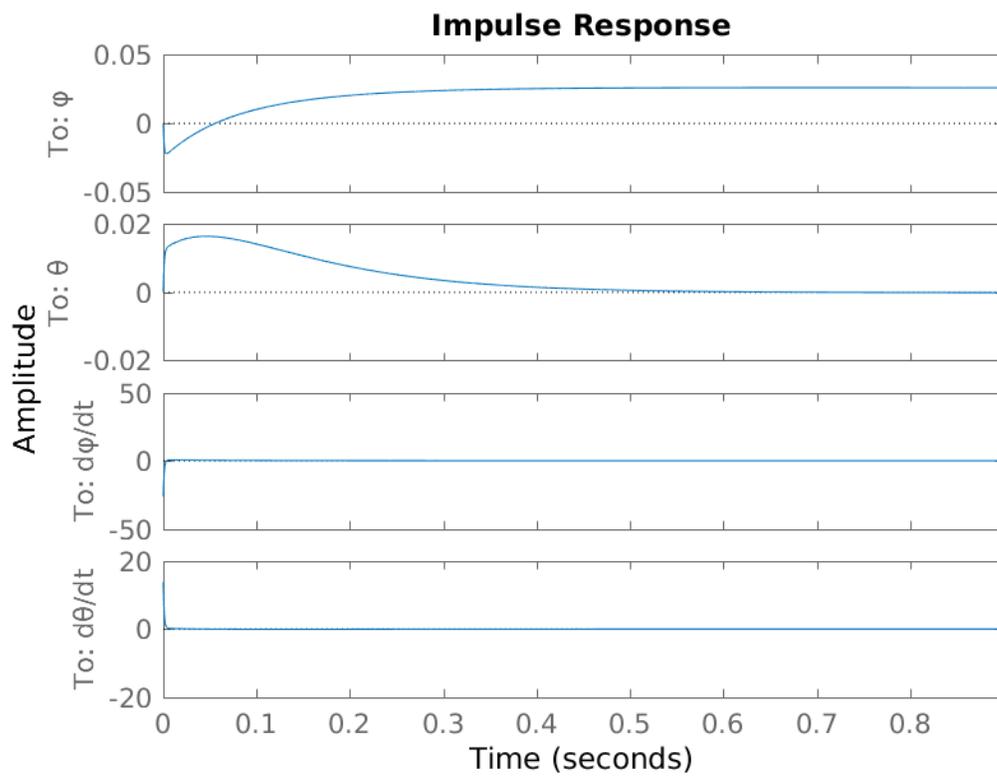


Fig. 4.2: LQR Design Simulation Results

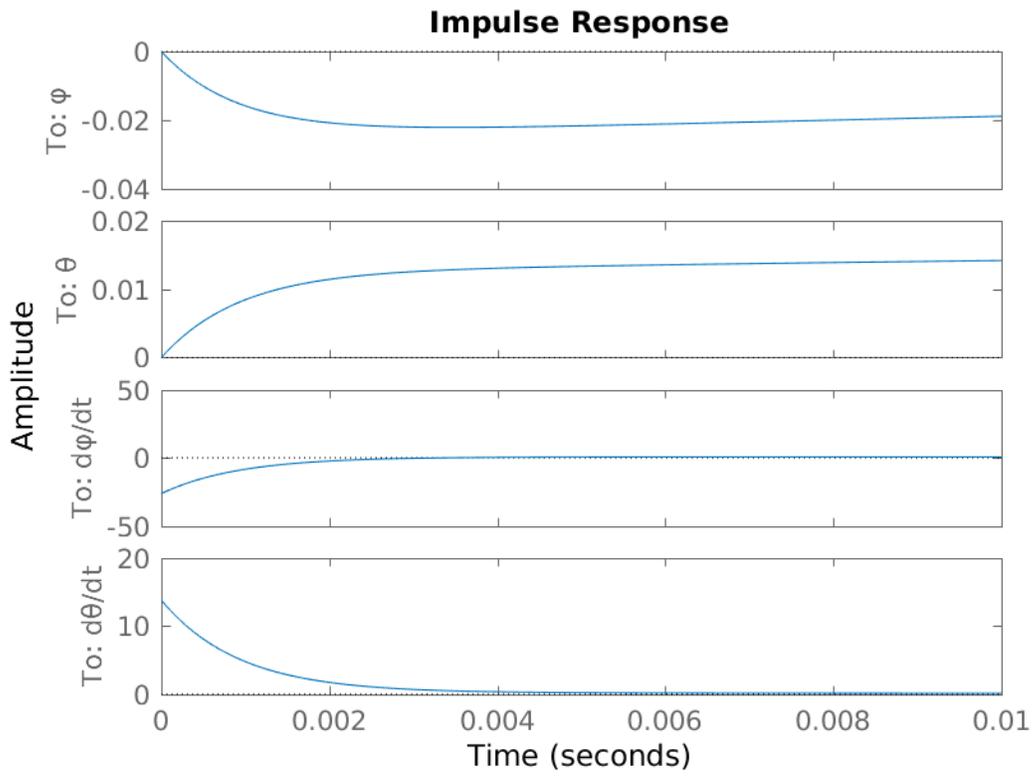


Fig. 4.3: LQR Design Zoomed Simulation Results

## 4.7 Further Features

### 4.7.1 Arduino Pretty Printing

For rapid deployment and testing the `scripts/lqr_design.m` also prints the scaled control law in the Arduino language. An example output can be seen in [Listing 4.9](#) and the script excerpt that implement this functionality in [Listing 4.8](#).

Listing 4.8: Arduino Code control law generator `scripts/lqr_design.m`  
[1]

```
disp("Control Law")
K_string = strcat("const float K[" , num2str(size(model.a,1)), "] = {");
for k = 1:size(K, 2)
    if ~(k == 1)
        K_string = strcat(K_string, ", ");
    end
    K_string = strcat(K_string, num2str(K_s(k)));
end
K_string = strcat(K_string, "};");
disp("K")
disp(K_string)
```

Listing 4.9: Arduino Code control law generator output

```
Control Law
K
const float K[4] = {0.017453, 8.4406, 0.1746, 0.35439}
```

### 4.7.2 Closed-Loop Poles

`scripts/lqr_design.m` also print out the closed-loop poles. An example output can be seen in [Listing 4.10](#).

Listing 4.10: Closed-loop printing output example

```
P =

1.0e+03 *

-1.0872
-0.0000
-0.0083
-0.0178
```

### 4.7.3 Full-compensator design

In `scripts/lqr_design.m` the full compensator design flow is implemented but since it's not being implemented it's was left out of this documentation.



---

## Controller Implementation

---

The implementation of the controller system was done using Arduino IDE. The functionality was separated into files within the Arduino IDE project. *ZumoIMU* and *ZumoEncoders* files were already explained in the sections *Inertial Management Unit* and *Encoders*.

### 5.1 SegwayLQR

#### 5.1.1 SegwayLQR API

##### Global Constants

```
const uint8_t samplingPeriodMS = 20  
    Sampling Period in ms  
const float samplingPeriod = samplingPeriodMS / 1000.0;  
    Sampling Period in s  
const float samplingFrequency = 1 / samplingPeriod  
    Sampling frequency  
const uint8_t statesNumber = 4  
    Number states
```

##### Global Variables

```
float angularPositionLP = 0  
    Low pass filter angular Position  
float angularPosition = 0  
    Zumo's angular position  
float correctedAngularPosition = 0  
    Corrected angular position
```

float **angularSpeed** = 0

Zumo's angular speed

float **motorAngularPosition** = 0

Motor's angular position

float **motorAngularSpeed** = 0

Motor's angular speed

int32\_t **speed**

PWM signal applied to the motor's driver 400 is 100% cycle and -400 is 100% but inverse direction

Zumo32U4ButtonA **buttonA**

A button of the zumo board

*Zumo32U4Motors* **motors**

Zumo robot's motors

## Functions

**class SegwayLQR**

void **setup** ()

Segway's setup function.

void **loop** ()

Segway's loop function.

void **setActuators** ()

Set the values to the actuators.

### 5.1.2 SegwayLQR Details

`src/SegwayLQR/SegwayLQR.ino` features the main loop and setup functions of the Arduino project. Listing 5.1 shows the implementation of the setup function. Firstly, it setups the IMU by calling `ZumoIMU::setupIMU()`. Then, calibrates the IMU's gyro by calling `ZumoIMU::calibrateGyro()`. After calibrating it starts a loop of sampling the gyro as frequently as possible and the accelerometer every `samplingPeriod`. When the `buttonA` is pressed the loop is exited and before starts executing `SegwayLQR::loop()` the encoders counters are cleared by calling `ZumoEncoders::clearEncoders()`.

Listing 5.1: Setup function

```
/**
 * Setup Function
 */
void setup() {
  Wire.begin();

  Serial.begin(115200);

  // Setup the IMU
  setupIMU();

  // Calibrate the IMU (obtain the offset)
  calibrateGyro();
```

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```

// Display the angle until the user presses A.
while (!buttonA.getSingleDebounceRelease()) {
    // Update the angle using the gyro as often as possible.
    sampleGyro();

    // Sample accelerometer every sampling period
    static uint8_t lastCorrectionTime = 0;
    uint8_t m = millis();
    if ((uint8_t)(m - lastCorrectionTime) >= samplingPeriodMS)
    {
        lastCorrectionTime = m;
        sampleAccelerometer();
    }
}
delay(500);
clearEncoders();
}

```

Listing 5.2 shows the loop function's code. Basically does the same as in the loop in `SegwayLQR::setup()` but every `samplingPeriod` it;

1. `ZumoIMU::sampleAccelerometer()` to obtain the corrected estimation of the Zumo's angle and angular speed, as explained in *Inertial Management Unit*.
2. `ZumoEncoders::sampleEncoders()` to obtain encoders position and speed, as explained in *Encoders*.
3. `SegwayLQR::setActuators()` calculates the new speed to be set based on the current state variables' state and the LQR designed control law.

Listing 5.2: Loop function

```

/**
 * Main loop Function
 */
void loop() {
    // Update the angle using the gyro as often as possible.
    sampleGyro();

    // Every 20 ms (50 Hz), correct the angle using the
    // accelerometer, print it, and set the motor speeds.
    static byte lastCorrectionTime = 0;
    byte m = millis();
    if ((byte)(m - lastCorrectionTime) >= 20)
    {
        lastCorrectionTime = m;
        sampleAccelerometer();
        sampleEncoders();
        setActuators();
    }
}

```

Listing 5.3 shows the `SegwayLQR::setActuators()` function's code. As a security measure when the angle is greater than  $45^\circ$  the speed is set to zero. Furthermore, the angle is corrected by the deviation of the COM from the actual horizontal center of the Zumo32u4. Finally the `LQR::lqr()` is called to apply the control law and generate the input of the system.

Listing 5.3: Set actuators function

```

/**
 * Control the actuators
 */
void setActuators() {
    const float targetAngle = 1.45;

    if (abs(angularPosition) > 45) {
        // If the robot is tilted more than 45 degrees, it is
        // probably going to fall over. Stop the motors to prevent
        // it from running away.
        speed = 0;
    } else {
        correctedAngularPosition = angularPosition - targetAngle;
        lqr();
        speed = constrain(speed, -400, 400);
    }

    motors.setSpeeds(speed, speed);
}

```

## 5.2 LQR

### 5.2.1 LQR API

**class** LQR

**const** float **K**[statesNumber] = {0.55192, 8.9867, 0.194, 0.39237}  
Control Law

**const** float **scaleConst** = 2.5  
Correction factor gain factor

void **lqr** ()  
Apply LQR control law

### 5.2.2 LQR Details

Listing 5.4 shows how the `LQR::lqr()` is implemented.

Listing 5.4: Apply LQR designed control law

```

/**
 * LQR control law
 */
void lqr() {
    speed = 0;

    speed -= motorAngularPosition * K[0];
    speed -= correctedAngularPosition * K[1];
    speed -= motorAngularSpeed * K[2];
}

```

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```
speed -= angularSpeed * K[3];  
speed = speed*scaleConst;  
}
```

---

**Note:**

- The K values are multiplied by  $-1$  in according to Fig. 4.1.
  - An additional scale factor,  $scaleConst = 2.5$ , is introduce to compensate;
    - Possible deviation of the actual Stall Torque with load.
    - Bad estimation of the  $\beta_m$  and  $\beta_\gamma$  values.
-



---

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